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EFFECT OF MOTION OF A GAS MEDIUM ON THE ACCURACY OF MEASUREMENT BY ACOUSTIC METHODS

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The effect of velocity and direction of motion of a gas medium on the propagation time of an acoustic signal is treated in the regions of unidirectional radiation and echo ranging.

Acoustic methods of measuring sizes and relative velocities of motion of objects are noted by their simplicity and high accuracy. They have found particularly wide application in oceanic studies [1]. Studies have recently appeared, devoted to applying these acoustic methods to solve similar problems in a gas medium. To this group belong problems such as the measurement of distance [2], noninertial temperature measurement of a gas medium [3], measurement of velocity and direction of displaced air flows [4], etc.

It must be noted that these problems are more complicated for a gas medium than for a liquid medium. This is primarily related to the fact that under real conditions a gas medium is quite mobile, the interference level is significantly higher in it, and the damping of acoustic waves is substantially higher [2]. For example, in a marine medium and for a sound velocity of approximately 1500 m/sec the flow velocity can reach values on the order of 1.5-2 m/sec, which can be neglected in many cases, with approximate equations used in the calculations. For a gas medium these equations are not valid under real conditions, since for a sound velocity of 340 m/sec the wind velocity can reach dozens of m/sec.

Approximate equations are derived in [2], devoted to these problems, which in the authors' opinion are valid for gas flow velocities up to 10 m/sec. However, the calculations presented below show that even in this velocity range of gas medium motion the measurement error for certain angles between the wind direction and the direction of sound emission is quite large.

We carried out a rigorous calculation of the propagation time of a sound wave for arbitrary values of the velocity and the directions of motion of the gas medium in the case of unidirectional sound emission and for the echo ranging regime, i.e., for the case in which sound

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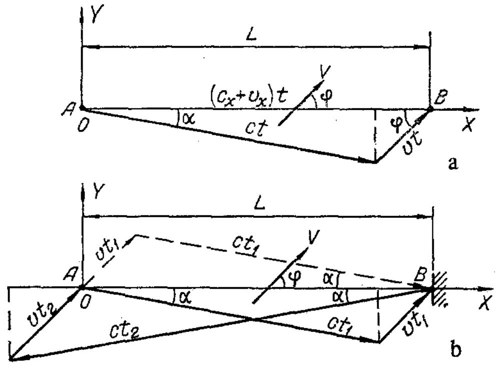

Fig. 1. For the calculation of the sound propagation time in a gas medium with unidirectional radiation (a) and echo ranging (b).

traverses a measurable distance twice, in the direct and opposite directions.

The regime of unidirectional sound emission is the most unfavorable from the point of view of amount of effect of medium motion on the measurement accuracy. We estimate the relative measurement error for this case.

Let there be an emitter at point A, a sound detector at point B, and let the medium in the portion AB move with velocity v at an angle φ to the OX axis (the coordinate system YOX lies in the reference plane of the vector C). The medium displacement at all points of the sounded space is assumed to be uniform and unidirectional. This assumption is usually valid for small volumes of the sounded space. The acoustic waves propagating from point A with velocity c will then behave linearly on the average, and waves are incident at the point B, emitted not in the direction of acoustic axis of the measurement channel, but at an angle α to it, as shown in Fig. 1a.

The following expression is valid for the scheme in Fig. 1a:

$$L=(c_x+v_x)t=ct\cos\alpha+vt\cos\varphi. \tag{1}$$

Considering similar triangles in Fig. 1a, it follows that

$$ct\sin\alpha=vt\sin\varphi, \tag{2}$$

$$\sin\alpha=\frac{v}{c}\sin\varphi, \tag{3}$$

$$\cos\alpha=\sqrt{1-\left(\frac{v}{c}\sin\varphi\right)^2}. \tag{4}$$

Taking these relations into account, we obtain

$$L=ct\sqrt{1-\left(\frac{v}{c}\sin\varphi\right)^2}+vt\cos\varphi. \tag{5}$$

The sound traversing time of the measured distance is then

$$t=\frac{L}{c\sqrt{1-\left(\frac{v}{c}\sin\varphi\right)^2}+v\cos\varphi}. \tag{6}$$

Without account of the medium motion this time is found from the relation

$$t_0=L/c. \tag{7}$$

The absolute measurement error in the case of medium motion is then

$$\Delta t = t - t_0 = \frac{L}{c}\left(\frac{1}{\sqrt{1-\left(\frac{v}{c}\sin\varphi\right)^2}+\frac{v}{c}\cos\varphi}-1\right). \tag{8}$$

Thus, in the case of arbitrary direction of the velocity vector of medium motion the relative measurement error for unidirectional radiation is, obviously,

$$\delta = \frac{t-t_0}{t_0} = \left(\frac{1}{\sqrt{1-a^2\sin^2\varphi}+a\cos\varphi}-1\right), \tag{9}$$

where a = v/c and the angle α (the leading angle of the reference acoustic ray) is, according to relationship (3),

$$\alpha = \arcsin(a\sin\varphi). \tag{10}$$

For medium motion along the axis of the measuring channel (longitudinal medium motion) we have two cases. In the first case, when the directions of the vectors V and C coincide (φ = 0), expression (9) acquires the form

$$\delta = \frac{1}{1+a} - 1. \tag{11}$$

In the second case, when the vectors V and C are oppositely directed ($\varphi = \pi$),

$$\delta = \frac{1}{1-a} - 1. \tag{12}$$

For transverse motion of the medium ($\varphi = \pi/2$ or $\varphi = 3\pi/2$)

$$\delta = \frac{1}{\sqrt{1-a^2}} - 1. \tag{13}$$

The data obtained refine substantially the results derived in [2]. In particular, relationship (10) establishes a connection between the quantity α, the advance drift angle of the acoustic beam, and the direction of motion of the medium (the angle φ), while expressions (11) and (12) show that the measurement errors of sound propagation time for opposite and wake motions of the medium are not equal, while for opposite flow the measurement error is larger, and for the wake direction it is smaller.

Using relation (10) and knowing the maximum flow velocity, one can formulate requirements concerning the directivity diagram of electroacoustic transformers. Thus, for a wind velocity of 30 m/sec the angle α reaches values on the order of 5°. This implies that for measurements in an air medium with a high wind velocity the angle 2β of the directivity diagram of electroacoustic transformers must be larger than 2α = 10°.

Comparison of expressions (11), (12), and (13) shows that most of the error is due to longitudinal motion of the medium, while the transverse one is negligibly small. If, for example, the medium velocity is 10 m/sec, then by (11) and (12) the error is around 3%, while by (13) the error is around 0.05%.

Consider the echo ranging regime. In this case the effect of medium motion on the measurement accuracy is substantially reduced, but total error compensation does not occur.

The sound emitter and detector are located at the point A, while at the point B at a distance L from it one places a planar, reflecting obstacle, perpendicular to the line AB (Fig. 1b). In this case, as well as in the one described above, the acoustic waves propagating from the point A toward the reflecting surface will be supported by the medium, and the wave will be incident at point B, emitted under angle α to the line AB. Reaching the planar obstacle, the sound wave is reflected from it, according to the reflection law, under the same angle, and moves toward the detector, anticipating a deflection.

The propagation time t_1 of the acoustic wave toward the obstacle is determined according to (6). Similarly the time t_2 for the case of wave propagation from the obstacle is determined

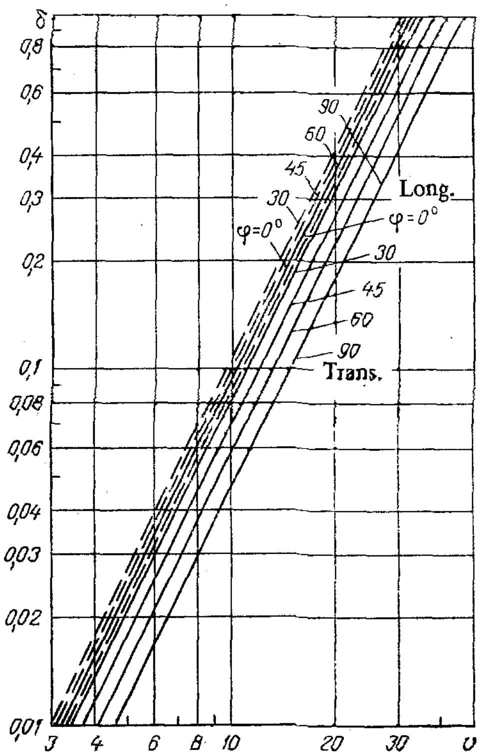

Fig. 2. Relative variation of the sound propagation time as a function of velocity and wind direction with echo ranging of a planar obstacle. δ, %; v, m/sec.

by the expression

$$t_2 = \frac{L}{c\sqrt{1 - \left(\frac{v}{c}\sin\varphi\right)^2} - v\cos\varphi}. \tag{14}$$

Using (6) and (14), we find after simple transformations

$$t = t_1 + t_2 = \frac{2L}{c}\,\frac{\sqrt{1 - a^2\sin^2\varphi}}{1 - a^2}. \tag{15}$$

For an immobile sound-conducting medium

$$t_0 = 2L/c. \tag{16}$$

The relative measurement error in a moving medium in the echo ranging region is then

$$\delta = \frac{\sqrt{1 - a^2\sin^2\varphi}}{1 - a^2} - 1. \tag{17}$$

In the case of longitudinal medium motion ($\varphi = 0$ or $\varphi = \pi$)

$$\delta = \frac{1}{1 - a^2} - 1, \tag{18}$$

and for transverse motion ($\varphi = \pi/2$ or $\varphi = 3\pi/2$)

$$\delta = \frac{1}{\sqrt{1 - a^2}} - 1. \tag{19}$$

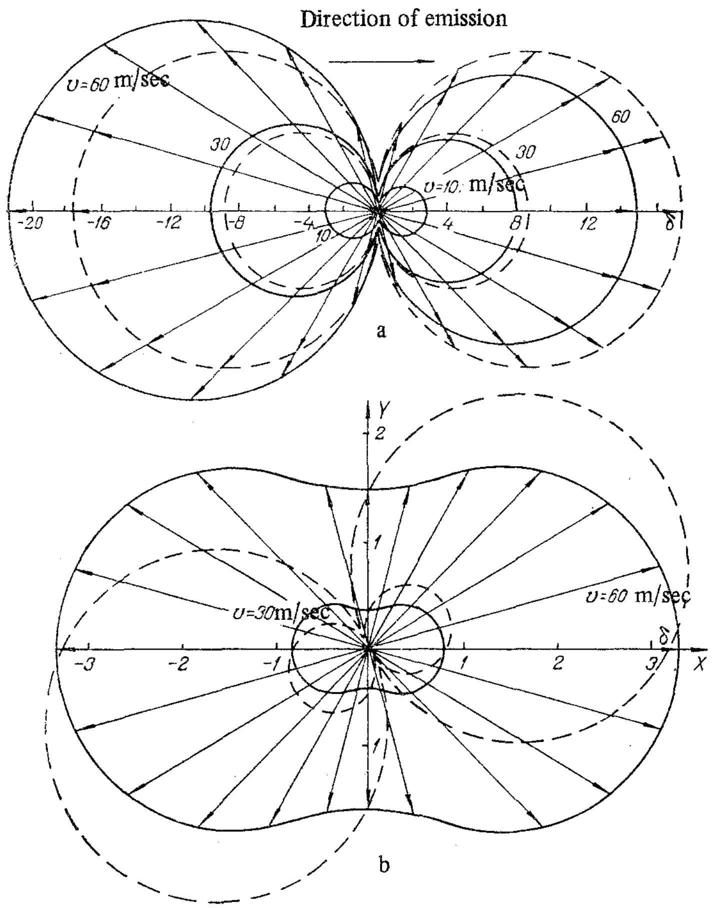

Fig. 3. Diagrams of the relative variation of the sound propagation time for different values and directions of the wind velocity for unidirectional emission (a) and echo ranging (b).

Comparing expressions (13) and (19), it is seen that they fully coincide, and this implies that transverse medium motion affects both measurement regimes equally.

Figure 2 shows the dependence of the measurement error of the time of flight at 0°C on the wind velocity for a planar obstacle echo ranging for longitudinal and transverse directions of motion of gas flow and angles φ = 30, 45, and 60°, The solid lines show the dependences obtained according to Eq. (17), and the dashed lines show, for comparison, the similar dependences, constructed by expressions (5-20), derived in [2]. It is seen from Fig. 2 that in the cases of longitudinal and transverse motion of a gas medium the measurement error of the propagation time of acoustic waves is practically the same, and the corresponding straight lines in Fig. 2 coincide. For other directions of medium motion the error values calculated by these dependences for the same values of wind velocity differ substantially. Thus, for example, for wind directed at 30° to the acoustic axis the error in determining the propagation time of sound oscillations differs in both cases by 21%, while for angles of 45 and 60° the difference consists of 29 and 33%, respectively.

Figure 3 shows angular diagrams of the relative measurement error of the sound propagation time in a gas medium as a function of the direction of its motion with velocities of 10, 30, and 60 m/sec in the regimes of unidirectional emission (Fig. 3a) and echo ranging (Fig. 3b). The error values are marked in the scale shown in the figure by lines corresponding to certain angles φ in the region from 0 to 360°. The lines are drawn for each 15°, and the angular diagrams calculated by expressions (9) and (17) are shown by solid lines, while the diagrams constructed by the data of [2] are shown by dashed lines.

As is seen, the angular diagrams calculated from Eq. (9), unlike the diagrams constructed from the data of [2], are nonsymmetric. The physical meaning of the result obtained consists in that for opposite medium motion the participation time of the wind in the distortion of the acoustic field is larger than for wake.

This fact is decisive in the formation mechanism of the error in echo ranging. At first glance, due to sound propagation in two mutually opposite directions total compensation must be observed in the time variation error. This, for example, follows from the symmetry of the diagram shown in Fig. 3a by dashed lines. However, this does not take place, since for sound propagation in one of the wind directions it is in the wake direction, while for propagation in the opposite direction it occurs in the opposite direction (see Fig. 1b). As a result the time increments, opposite in sign for sound propagation in the direct and opposite directions, are not equal in absolute value, and their difference creates a measurement error.

Thus, the equation given in [2] for the case of unidirectional emission is incorrect. Indeed, according to the diagram shown in Fig. 3a by the dashed line, the measurement errors of the sound propagation time for mutually opposed wind directions are identical, from which it follows that in the echo ranging regime and for any directions and wind velocities a variation of the sound propagation time must not occur due to mutual compensation of time increments during direct and opposite sound propagation. Practically this is not observed, and measurement errors are inevitably generated.

The angular diagram of the measurement error of the sound propagation time in the echo ranging regime is shown in Fig. 3b by solid lines. It is symmetric with respect to the OX and OY coordinate axes, and is elongated in the direction of the acoustic axis of the measuring channel. This agrees with the data obtained for the case of unidirectional emission. Indeed, in replacing the oppositely directed medium motion by the wake, the diagram shown in Fig. 3a by the solid line is mirror reflected. As a result the error measurements in replacing the wind direction by the opposite one remain identical in the echo ranging regime due to sound propagation in the direct and opposite directions.

As to the diagrams shown in Fig. 3b by dashed lines, their shape and rotation around the measurement axis are deprived of physical meaning, and indicate the error in the starting premises assumed in deriving the equations in [2].

Thus, the calculations show that wind interference in acoustic measurements is insignificant in the echo ranging regime, and in most practical cases does not need to be accounted for. In unidirectional sounding these interferences cannot be neglected, and to take them into account one must use Eqs. (6) and (8).

We also note that the relations obtained in the present study can serve as a basis for developing acoustic measurements of the wind velocity, having both independent value and predestined for the use of technical means, guaranteeing the independence of measurement results on wind interference.

NOTATION

c, sound velocity in a gas medium; v, velocity of motion of the medium (the wind velocity); L, length of the acoustic base; t, time; a, wind to sound velocity ratio; α, angle of allowance of the acoustic ray (the angle between the linear acoustic base in the direction of sound emission and the sound velocity vector); φ, angle between the linear acoustic base in the direction of sound emission and the sound velocity vector; C, sound velocity vector; V, wind velocity vector; 2β, solid angle of the directivity diagram of the electroacoustic transformer.

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